Quantum mechanics:
Hilbert space, spectrum of self-adjoint operators.
Basic principles of quantum mechanics: states, observables and expectation values, time evolution (Schrodinger equation and Heisenberg equation), canonical quantization, density matrix.
Exact solutions: harmonic oscillator, 3d particle in Coulomb field, particle in electric-magnetic field.
Symmetry in quantum mechanics: symmetry representation on Hilbert space, representation theory of permutation group and SU(2) group, addition of angular momentum, spin of electrons, identical particles, and atomic structure.
Approximation method: variational method, time independent perturbation theory, time dependent perturbation theory, WKB method.
Scattering theory: Lippman-Schwinger equation, scattering amplitude, Born approximation, partial wave method, computation of phase shift, time dependent perturbation theory.

Reference: J.J. Sakurai Mordern quantum mechanics

General relativity:
Basic computations: Tensor calculous, geodesic, Einstein equations, symmetry
Exact solutions: Schwarzschild metric, Reissner–Nordström metric, Kerr metric
Global structure: Causal structure, Cauchy surface, Penrose-Hawking theorem of singularity

Reference: R.Ward General relativity

Quantum field theory:
Representation of Poincare group
Perturbation theory: Canonical quantization of spin 0,1/2, 1 particles, Scattering amplitude and Feynman diagrams
Regularization and Renormalization, Renormalization group.
Symmetry and anomalies: Ward identity, spontaneous symmetry breaking, Higgs mechanism, anomalies
Extended objects: monopoles, instantons

Reference: S. Weinberg The quantum theory of fields Vol 1,2,3