Syllabus on Computational and Applied Mathematics

Interpolation and approximation:

Trigonometric interpolation and approximation, fast Fourier transform; approximations by rational functions; polynomial and spline interpolations and approximation; least-squares approximation.

Nonlinear equation solvers:

Convergence of iterative methods (bisection, Newton's method, quasi-Newton’s methods and fixed-point methods) for both scalar equations and systems, finding roots of polynomials.

Linear systems and eigenvalue problems:

Classical and modern iterative method for linear systems and eigenvalue problems, condition number and singular value decomposition, iterative methods for large sparse system of linear equations

Numerical solutions of ordinary differential equations:

Single step methods and multi-step methods, stability, accuracy and convergence; absolute stability, long time behavior; numerical methods for stiff ODE’s.

Numerical solutions of partial differential equations:

Finite difference method, finite element method and spectral method: stability, accuracy and convergence, Lax equivalence theorem.

Mathematical modeling, simulation, and applied analysis:

Scaling behavior and asymptotics analysis, stationary phase analysis, boundary layer analysis, qualitative and quantitative analysis of mathematical models, Monte-Carlo method.

Linear and nonlinear programming:

Simplex method, interior method, penalty method, Newton's method, homotopy method and fixed point method, dynamic programming.

References:

[1] C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, 1999.

[2] C. de Boor and S.D. Conte, *Elementary Numerical Analysis, an algorithmic approach,* McGraw-Hill, 2000.

[3] G.H. Golub and C.F. van Loan, *Matrix Computations, third edition*, Johns Hopkins University Press, 1996.

[4] E. Hairer, P. Syvert and G. Wanner, *Solving Ordinary Differential Equations,* Springer, 1993.

[5] B. Gustafsson, H.-O. Kreiss and J. Oliger, *Time Dependent Problems and Difference Methods*, John Wiley Sons, 1995.

[6] J. Keener, "*Principles of Applied Mathematics*", Addison-Wesley, 1988.

[7] Lloyd N. Trefethen and David Bau, *Numerical linear algebra*, SIAM, 1997.

[8] Susanne Brenner and Ridgway Scott, *The Mathematical Theory of Finite Element Methods*, Springer, 2010.

[9] F.Y.M. Wan, *Introduction to Calculus of Variations and Its Applications*, Chapman & Hall, 1995