Syllabus on Analysis and Differential Equations

Real Analysis:

- Convergence theorems for integrals, Borel measure, Riesz representation theorem
- L^p space, Duality of L^p space, Jensen inequality
- Lebesgue differentiation theorem, Fubini theorem, Hilbert space
- Complex measures of bounded variation, Radon-Nikodym theorem.
- Hahn-Banah Theorem, open mapping theorem, uniform boundedness theorem, closed graph theorem.
- Basic properties of compact operators, Riesz-Fredholm Theory, spectrum of compact operators.
- Fourier series, Fourier transform, convolution.

References: Rudin: Real and Complex Analysis; Stein and Shakarchi: Real analysis; Stein and Shakarchi: Fourier Analysis.

Complex Analysis:

- Holomorphic and meromorphic functions
- Conformal maps, linear fractional transformations, Schwarz's lemma

- Complex integrals: Cauchy's theorem, Cauchy integral formula, residues
- Harmonic functions: the mean value property; the reflection principle; Dirichlet's problem
- Series and product developments: Laurent series, partial fractions expansions, and canonical products
- Special functions: the Gamma function, the zeta functions and elliptic functions
- Basics of Riemann surfaces
- Riemann mapping theorem, Picard theorems.

References: Ahlfors: Complex Analysis (3rd edition)

Differential Equations:

- Existence and uniqueness theorems for solutions of ODE; explicit solutions of simple equations; self-adjoint boundary value problems on finite intervals; critical points, phase space, stability analysis.
- First order partial differential equations, linear and quasi-linear PDE.
- Phase plane analysis, Burgers equation, Hamilton-Jacobi equation.

- Potential equations: Green functions and existence of solutions of Dirichlet problem, harmonic functions, maximal principle and applications, existence of solutions of Neumann's problem.
- Heat equation, Dirichlet problem, fundamental solutions
- Wave equations: initial condition and boundary condition, well-posedness, Sturm-Liouville eigenvalue problem, energy functional method, uniqueness and stability of solutions
- Distributions, Sobolev embedding theorem.

References: V. I. Arnold: Mathematical Methods of Classical Mechanics; Craig Evans: Partial Differential Equations.