Syllabus on Algebra and Number Theory

Algebra:

- Group theory: Sylow theorems, p-groups, solvable groups, free groups.
- Rings and modules: tensor products, determinants, Jordan canonical form, PID's, UFD's, polynomials rings.
- Field theory: splitting fields, separable and inseparable extensions.
- Galois theory: Fundamental theorems of Galois theory, finite fields, cyclotomic fields.
- Homological algebra: exact sequences, splittings, snake and five lemmas, projective, injective, and flat modules, complexes, (co)homology.
- Commutative ring: localizations, Hilbert’s basis theorem, integral extensions, radicals of ideals, Zariski topology and Hilbert’s Nullstellensatz, Dedekind rings, DVRs.
- Representations of Finite Groups: character theory, induced representations, structure of the group ring.
- Basics of Lie groups and Lie algebras: exponential map, nilpotent and semi-simple Lie algebras and Lie groups.

References: Dummit and Foote: Abstract Algebra, 2nd edition; Serre: Representations of Finite Groups; Fulton-Harris: Representation Theory:

A First Course (Graduate Texts in Mathematics/Readings in Mathematics); Serge Lang: Algebra.

Number Theory:

Factorization and the primes; congruences; quadratic residues and reciprocity; continued fractions and approximations; eta functions; zeta functions;

Number fields; unique factorization of ideals; finiteness of class group; structure of unit group; Frobenius elements; local fields; ramification; weak approximation.

References: Z.I. Borevich, Igor Shafarevich, Number Theory; Serge Lang, Algebraic Number Theory